Ma104: Mathematics for Business Major

Tutorial 3 (Answer)

1. **Show that the line represented by 4x + 8y = 10 and 2x = 12 – 4y are parallel.**

We solve each equation for y to see that the lines are distinct and that their slopes are equal.

$4x + 8y = 10$	$2x = 12 - 4y$
$8y = -4x + 10$	$4y = -2x + 12$
$y = -\frac{1}{2}x + \frac{5}{4}$	$y = -\frac{1}{2}x + 3$

Since the values of b in these equations are different, the lines are distinct. Since the slope of each line is $-\frac{1}{2}$, they are parallel. ■

2. **Show that the lines represented by 4x + 8y = 10 and 4x – 2y = 21 are perpendicular.**

We solve each equation for y to see that the slopes of their straight-line graphs are negative reciprocals.

$4x + 8y = 10$	$4x - 2y = 21$
$8y = -4x + 10$	$-2y = -4x + 21$
$y = -\frac{1}{2}x + \frac{5}{4}$	$y = 2x - \frac{21}{2}$

Since the slopes of the lines are $-\frac{1}{2}$ and 2 (which are negative reciprocals), the lines are perpendicular. ■

3. **Use the slope-intercept form to write the equation of the line passing through the point P(-2, 5) and parallel to line y = 8x –2.**

Since the slope of the line given by $y = 8x - 2$ is the coefficient of x, the slope of the line is 8. Thus, the line represented by the desired equation must also have a slope of 8, because it is parallel to the graph of $y = 8x - 2$.

We substitute -2 for x, 5 for y, and 8 for m in the slope–intercept form and solve for b.

$$y = mx + b$$

$5 = 8(-2) + b$	Substitute 5 for y, 8 for m, and -2 for x.
$5 = -16 + b$	Simplify.
$21 = b$	Add 16 to both sides.

Since $m = 8$ and $b = 21$, the equation of the desired line is $y = 8x + 21$. ■

4. **A television program director must schedule comedy skits and musical number for prime-time variety shows. Each comedy skit requires 2 hours of rehearsal time, cost \$3,000, and brings in \$20,000 from the shows' sponsors. Each musical number requires 1 hour of rehearsal time, cost \$6,000 and generates \$12,000. If 250 hours are available for rehearsal and \$600,000 is budgeted for comedy and music, how many of each type should be produced, in order to maximum income? Find the maximum income.**

We can let x represent the number of comedy skits and y the number of musical numbers to be scheduled. We then construct a table with information about rehearsal time, production cost, and income generated.

	Comedy	Musical	Available
Rehearsal time (hours)	2	1	250
Cost (in \$1000s)	3	6	600
Generated income (in \$1000s)	20	12	

Next we write the objective function. Since each of the x comedy skits generates 20 thousand dollars, the income generated by the comedy skits is $20x$ thousand dollars. The musical numbers produce $12y$ thousand dollars. The objective function to be maximized is

$$V = 20x + 12y$$

Then we write the constraints. Since there are limits on rehearsal time and budget, there is a constraint for each. Note that neither x nor y can be negative.

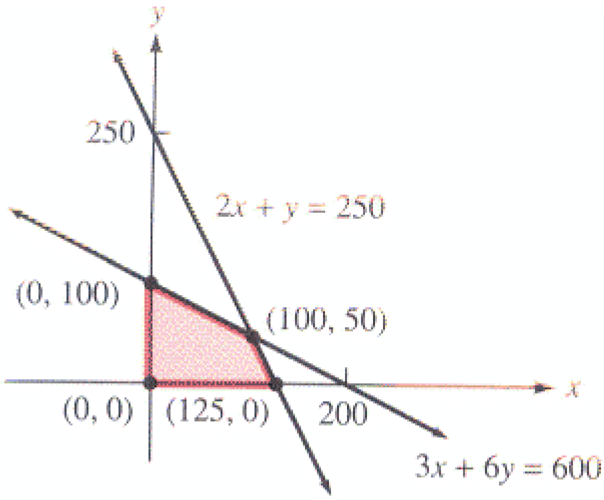
Constraint on rehearsal time	$2x + y \leq 250$
Constraint on cost	$3x + 6y \leq 600$
Nonnegative constraints	$x \geq 0,\ y \geq 0$

We graph the inequalities to find the feasible region shown in Figure 10-16 and find the coordinates of each corner point.

The coordinates of the corner points of the feasible region are (0, 0), (0, 100), (100, 50), and (125, 0). To find the maximum income, we substitute each pair of coordinates into the objective function.

Corner point	$V = 20x + 12y$
(0, 0)	$V = 20(0) + 12(0) = 0$
(0, 100)	$V = 20(0) + 12(100) = 1200$
(100, 50)	$V = 20(100) + 12(50) = 2600$
(125, 0)	$V = 20(125) + 12(0) = 2500$

Maximum income occurs if 100 comedy skits and 50 musical numbers are scheduled. The maximum income will be 2600 thousand dollars, or \$2,600,000.


y
250
2x + y = 250
(0, 100)
(100, 50)
x
(0, 0)
(125, 0)
200
3x + 6y = 600