Ma104: Mathematics for Business Major

Tutorial 2 (Answer)

1. A sofa and matching chair are on sales for $777. If the list price was $925, find the percent of markdown.

In this case, $777 is the sale price, $925 is the regular price, and the markdown is the product of $925 and the percent of markdown.

We can let r represent the percent of markdown, expressed as a decimal. We substitute $777 for the sale price and $925 for the regular price in the formula

Sale price	=	regular price	−	markdown.
777	=	925	−	$r \cdot 925$

$777 = 925 - 925r$

$-148 = -925r$ Subtract 925 from both sides.

$0.16 = r$ Divide both sides by −925.

The percent of markdown is 16%.

2. A college foundation owns stock in IBC (selling at $54 per share), GS (selling at $65 per share), ATB (selling at $105 per share). The foundation owns equal shares of GS and IBC, but five times as many shares of ATB. In this portfolio is worth $450,800, how many shares of each type does the foundation own?

Analysis The value of the IBC stock plus the value of the GS stock plus the value of the ATB stock must equal $450,800.

- If x represents the number of shares of IBC, then $54x is the value of that stock.
- Since the foundation has equal numbers of shares of GS as IBC, x also represents the number of shares of GS. The value of this stock is $65x.
- Since the foundation owns five times as many shares of ATB, it owns $5x$ shares of ATB. The value of this stock is $105(5x).

We set the sum of these values equal to $450,800.

Solution We let x represent the number of shares of IBC.
Then x also represents the number of shares of GS.
Then $5x$ represents the number of shares of ATB.

The value of IBC stock	+	the value of GS stock	+	the value of ATB stock	=	the total value of the stock.
$54x$	+	$65x$	+	$105(5x)$	=	450,800

$54x + 65x + 525x = 450{,}800$ $105(5x) = 525x$.

$644x = 450{,}800$ Combine terms.

$x = 700$ Divide both sides by 644.

The foundation owns 700 shares of IBC, 700 shares of GS, and 5(700), or 3500, shares of ABT.

3. **One machine has a setup cost of \$400 and a unit cost of \$1.50, and another machine has a setup cost of \$500 and a unit cost of \$1.25. Find the break point.**

Finding the break point One machine has a setup cost of \$400 and a unit cost of \$1.50, and another machine has a setup cost of \$500 and a unit cost of \$1.25. Find the break point.

If x represents the number of items to be manufactured, the cost C_1 using machine 1 is

$$C_1 = 400 + 1.5x$$

and the cost C_2 using machine 2 is

$$C_2 = 500 + 1.25x$$

The break point is the value of x when $C_1 = C_2$.

Let x represent the number of items to be manufactured.
Then $400 + 1.5x$ represents the cost using machine 1, and $500 + 1.25x$ represents the cost using machine 2.

The break point occurs when these two costs are equal.

The cost of using machine 1	=	the cost of using machine 2.
$400 + 1.5x$	=	$500 + 1.25x$

$1.5x = 100 + 1.25x$ Subtract 400 from both sides.

$0.25x = 100$ Subtract $1.25x$ from both sides.

$x = 400$ Divide both sides by 0.25.

The break point is 400 units.

4. **A professor has \$15,000 to invest for 1 year. He invests some at 8% and then rest at 7%. If his total profit from these investments is \$1,110, how much did he invest at each rate?**

We will add the interest from the 8% investment to the interest from the 7% investment and set the sum equal to the total interest earned.

For simple interest, the interest earned is computed by the formula $i = prt$, where p is the principal, i is the annual interest rate, and t is the length of time the principal is invested. Thus, if \$$x$ are invested at 8%, the interest earned is \$$0.08x$. If the remaining \$$(15{,}000 - x)$ is invested at 7%, the amount earned on that investment is \$$0.07(15{,}000 - x)$. The sum of these two amounts equals \$1110.

We can let x represent the number of dollars invested at 8%.
Then $15{,}000 - x$ represents the number of dollars invested at 7%.

The interest earned at 8%	+	the interest earned at 7%	=	the total interest.
$0.08x$	+	$0.07(15{,}000 - x)$	=	1110

$8x + 7(15{,}000 - x) = 111{,}000$ Multiply both sides by 100 to eliminate the decimals.

$8x + 105{,}000 - 7x = 111{,}000$ Use the distributive property to remove parentheses.

$x + 105{,}000 = 111{,}000$ Combine terms.

$x = 6000$ Subtract 105,000 from both sides.

$15{,}000 - x = 9000$

She invested \$6000 at 8% and \$9000 at 7%.

5. **Using the augment matrix and Gaussian elimination to solve the system of equation:**

2x – y + z = -3

x + y – z = 6

3x – y – z = 4

Start with the augmented matrix and interchange the first and second rows to get a 1 in the upper left position in the matrix:

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & -3 \\ 1 & 1 & -1 & 6 \\ 3 & -1 & -1 & 4 \end{array}\right] \quad \text{The augmented matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & -1 & 1 & -3 \\ 3 & -1 & -1 & 4 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

Now multiply the first row by -2 and add the result onto the second row. Multiply the first row by -3 and add the result onto the third row. These two steps eliminate the variable x from the second and third rows:

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -3 & 3 & -15 \\ 0 & -4 & 2 & -14 \end{array}\right] \quad \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array}$$

Multiply the second row by $-\frac{1}{3}$ to get 1 in the second position on the diagonal:

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 1 & -1 & 5 \\ 0 & -4 & 2 & -14 \end{array}\right] \quad -\tfrac{1}{3}R_2 \to R_2$$

Use the second row to eliminate the variable y from the first and third rows:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & -2 & 6 \end{array}\right] \quad \begin{array}{l} -1R_2 + R_1 \to R_1 \\ \\ 4R_2 + R_3 \to R_3 \end{array}$$

Multiply the third row by $-\frac{1}{2}$ to get a 1 in the third position on the diagonal:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad -\tfrac{1}{2}R_3 \to R_3$$

Use the third row to eliminate the variable z from the second row:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad R_3 + R_2 \to R_2$$

This last augmented matrix represents the system $x = 1, y = 2$, and $z = -3$. So the solution set to the system is $\{(1, 2, -3)\}$.