Functions and Graphs

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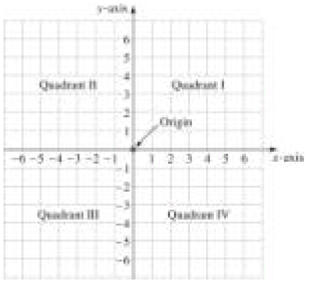
Ordered Pairs

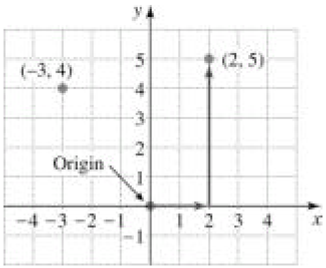
- All equations in two variables, such as y = mx + c, is satisfied only if we find a value of x_1 and a value of y_1 that make it true.
- These two values are called **Ordered Pairs** and denoted as (x_1, y_1).

Plotting Points

- The **Rectangular** and **Cartesian Coordinates System** consists of a horizontal number line, the **x-axis**, and a vertical line, the **y-axis**.
- The intersection of the axes is the **Origin**.
- The axes divide the coordinate plane, or the **xy-plane**, into four regions called the **Quadrants**.
- Locating a point in the rectangular coordinate system is referred to as **plotting** or **graphing** the point.

Cartesian Coordinates System





Intercepts

- The **x-intercept**, or **Abscissa** of a line is the point where the line cross the x-axis.
  - The value of x-intercepts is always (x_1, 0)
- The **y-intercept**, or **Ordinate** of a line is the point where the line cross the y-axis.
  - The value of y-intercepts is always (0, y_1)

Distance between Two Points

- Distance Formula
  - The distance between two Point $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example

- Consider the three point A = (-2, 1), B = (2, 3) and C = (3, 1).
  - Plot each point and form the triangle ABC.
  - Find the length of each side of the triangle.
  - Verify that the triangle is a right triangle.
  - Find the area of the triangle.

Midpoint of Line Segment

- Midpoint Formula
  - The Midpoint M = (x, y) of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example

- Find the midpoint of the line segment from $P_1(-5, 3)$ to $P_2(3, 1)$.

Graphing Lines by Plotting Points

- We know that the solution set to an equation is a line if the equation is linear in two variables – that is, there are two variables in the problem, and each variable only appears to a single power, is only in the numerator of fractions, is never under a radical symbol, and is not in the same term as the other variables.
- To graph lines by plotting points, first make a table of values and then plot each point.

Graphing Lines Using Intercepts

- Graphing line using intercept is to find the x- and y-intercepts, and since we know that the equation represents a straight line, draw a line between these two points.
- This method does not always work if the line through the origin (0, 0).

Slope

- The slope of a line is a measure of its steepness.
- A line with slope whose absolute value is very steep is steep, while a line with a slope whose absolute value is very steep is shallow.
- The formula for the slope *m* between two points (x_1, y_1) and (x_2, y_2) on a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of Parallel and Perpendicular Lines

- Two lines with slopes m_1 and m_2 are parallel if and only if
  - $m_1 = m_2$
- Two lines with slopes m_1 and m_2 are perpendicular if and only if
  - $m_1 \times m_2 = -1$

Formation of Equation

- There are four basic methods used to find a equation of a line from a description of it.
  - Two point Form
  - Point-Slope Form
  - Slope-Intercept Form
  - Intercept Form
- The equation in the form of Ax + By + C = 0 is called a **Standard Form**.

Comparison of Different Methods

Form	**Information Needed**	**Equation**
Two Points Form	Point: (x_1, y_1) Point: (x_2, y_2)	$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$
Point-Slope Form	Slope: m Point: (x_1, y_1)	$y-y_1=m(x-x_1)$
Slope-Intercept Form	Slope: m y-intercept: (0, c)	$y=mx+c$
Intercept Form	x-intercept (b, 0) y-intercept: (0, c)	$\frac{x}{b}+\frac{y}{c}=1$

Methods for Graphing a Linear Equation

Methods for Graphing a Linear Equation

1. Arbitrarily select some points that satisfy the equation, and draw a line through them.
2. Find the x- and y-intercepts (provided that they are not the origin), and draw a line through them.
3. Start at the y-intercept and use the slope to locate a second point, then draw a line through the two points.

Linear Inequality

- If A, B and C are real numbers with A and B are not zero, then
  - $Ax + By \le C$
  - $Ax + By \ge C$
  - $Ax + By < C$
  - $Ax + By > C$
- is called a **Linear Inequality**.

Graphing a Linear Inequality

Graphing a Linear Inequality

1. Solve the inequality for y, then graph $y = mx + b$.

$y > mx + b$ is satisfied above the line.

$y = mx + b$ is satisfied on the line itself.

$y < mx + b$ is satisfied below the line.

2. If the inequality involves x and not y, then graph the vertical line $x = k$.

$x > k$ is satisfied to the right of the line.

$x = k$ is satisfied on the line itself.

$x < k$ is satisfied to the left of the line.

Graphing Compound Inequalities

1. **Graph** the lines
   - Use a dashed line if the line **is not** to be included in the solution set (< or >).
   - Use a solid line if the line is to be included in the solution set (≤ or ≥).
2. The plane of graph is divided into two or more region by the lines. **Find a Test Point** for each region.
3. **Check** to see if the test point satisfy the inequality.
4. **Shade** the appropriate regions.

Example

- Graph the solution set to the compound inequality $2x + y \le 4$ and $x - 2y > 7$.

Solving Linear Systems by Graphing

- When solving a system of equations by graphing, it is very important for the graph to be as accurate as possible.
- Keep in mind that graph is next exact, and when you solve by graphing, you need to minimize the errors in the graph.
- After solved a system of equations by graph, MUST check the answer in both of the original equations.
- Remember that the point of intersection must lie on both lines to satisfy both equations.

Relation

- A Relation is a correspondence between two sets. If x and y are two elements in these set and if a relation exists between x and y, then we say that x corresponds to y or that y depends on x, and write $x \rightarrow y$. We may also write $x \rightarrow y$ as the ordered pair (x, y).

Functions

- Let X and Y be two non-empty set. A function from X into Y is a relation associates with each element of X exactly one element of Y.
- The set X is called the **Domain** of the function.
- For each element x in X, the corresponding element y in Y is called the **Value** of the function at x or the **Image** of x.
- The set of all images of the elements of the domain is called the **Range** of the function.

Notation of Function

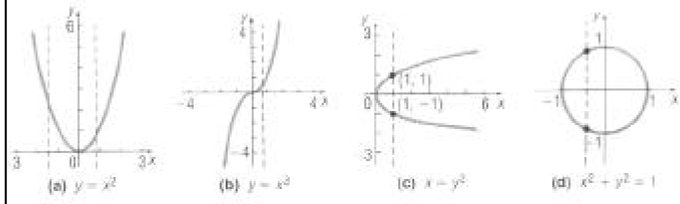
- f: $X \rightarrow Y$ indicate f is a function from X to Y.
- For a function y=f(x), the variable x is called the **Independent Variable** or **Argument** of the function because it can be assigned any of the permissible number from the domain. The variable y is called the **Dependent Variable** because its value depend on x.

Vertical-line Test

- A set of points in the xy-plane is the graph of a function if and only if every vertical intersects the graph in at most one point.

Example

- Which of the graphs below are graphs of functions?



Average Rate of Change

- If c is in the domain of a function y = f(x), the **Average Rate of Change** of *f* from c to x (for x ≠ c) is defined as

$$\text{Average Rate of Change} = \frac{\Delta x}{\Delta y} = \frac{f(x) - f(c)}{x - c}$$

- This expression is also called the **Different Quotient** of f at c.
- The Average Rate of Change of a function equals the slope of the secant line containing two points on its graph.

Average Rate of Change

- A function *f* is Increasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.
- A function *f* is Decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$
- A function *f* is Constant on an open interval I if, for any choice of x in I, the value of *f* (x) are equal.

Local Maximum and Minimum

- A function f has a **Local Maximum at c** if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) < f(c)$. We call $f(c)$ a **Local Maximum of *f***.
- A function f has a **Local Minimum at c** if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) > f(c)$. We call $f(c)$ a **Local Minimum of *f***.

Even and Odd Function

- Even function
  - A function f(x) is said to be an even function if $f(-x) = f(x)$.
- Odd function
  - A function f(x) is said to be an odd function if $f(-x) = -f(x)$.

Library of Functions

- Linear Functions
  - $f(x) = mx + c \;\; (\forall m, c \in R)$
- Constant Functions
  - $f(x) = c \;\; (\forall c \in R)$
- Identity Functions
  - $f(x) = x$
- Square Functions
  - $f(x) = x^2$

Library of Functions

- Cube Functions
  - $f(x) = x^3$
- Square Root Functions
  - $f(x) = \sqrt{x}$
- Reciprocal Functions
  - $f(x) = 1/x$
- Absolute Value Functions
  - $f(x) = |x|$

Operations on Functions

- The **Sum of *f* and g**, $f + g$ is defined by
  $(f + g)(x) = f(x) + g(x)$
- The **Difference of *f* and g**, $f - g$ is defined by
  $(f - g)(x) = f(x) - g(x)$
- The **Product of *f* and g**, fg is defined by
  $(fg)(x) = f(x) \bullet g(x)$
- The **Sum of *f* and g**, f / g is defined by
  $(f / g)(x) = f(x) / g(x)$

Composite Functions

- Given two functions *f* and g, the **Composite Function**, denoted by $f \circ g$ is defined by
  $(f \circ g)(x) = f(g(x))$
- The domain of $f \circ g$ is the set of all numbers x in the domain of g such that g(x) is in the domain of *f*.

Example

- Given $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:
  - $(f \circ g)(1)$
  - $(g \circ f)(1)$
  - $(f \circ f)(-2)$
  - $(g \circ g)(-1)$