

# Systems of Equations and Inequalities

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## System of Equations

- A set of equation is called a **System of Equations**.
- The solutions of a system of equations must satisfy every equation in the system.

## Solving Linear Systems using Substitution

1. Solve one of the equations for each variables
2. Substitute this expression into the other equation (the one that did not use in Step 1)
3. Solve the resulting equation
4. Substitute this answer back into your expression from Step 1
5. Check the answer in the original equations.

## Example

- Solve the system of equations:

$$3x + 2y = 6$$

$$2x - y = 5$$

## Solving Linear Systems using the Addition Method (Elimination Method)

1. **Write the equations in the same form** .
2. **Multiply** the equations by a numbers so that in the resulting equations one of the variables has opposite coefficients for the two equations.
3. **Add** the equation together term-by-term.
4. **Solve** the resulting equation.
5. **Substitute** this answer back into either original equation and solve for the remaining variables.
6. **Check** your answer in both of the original equations.

## Example

- Solve the system of equations:

$$3x - 2y = 5$$

$$7x + 5y = 2$$

## Solving Systems of Linear Equations in Three Variables

1. Choose two of the three original equations and, using addition or substitution, combine the equations so that one of the three variables is eliminated.
2. Choose another pair of the original equation and, using the same method as in Step 1, combine these equations to eliminate the same variable as you did in Step 1.

## Solving Systems of Linear Equations in Three Variables

3. Using the results obtained in Step 1 and Step 2, form a new system of two equations in two unknown and solve it using addition or substitution.
4. Substitute the value of the two variables obtained in Step 3 into one of the three original equations and solve for the value of the third variables.
5. Check the answer in all three of the original equation.

## Example

- Solve the system of equations:

$$3x + 2y - 3z = 4$$

$$x + 3y - 7z = -2$$

$$2x - y = 1$$

## Solving Linear Systems using Matrices

- The augmented matrix and Gaussian elimination can be used to solve a system of  $n$  linear equations in  $n$  unknowns for a natural number  $n$ .
- To write the augmented matrix, write each equation in standard form, always keeping the variables lined up vertically and constants on the right-hand side of the equation. If a variable is missing, you can either leave the spot for it empty, or include a zero for it.
- Copy the coefficients into the matrix, remembering the negative signs when appropriate.
- The position in the matrix tells which variable the number is a coefficient of.

## Steps for Solving Linear Systems using Matrices

1. Interchanging Row (this corresponds to writing the equations in a different order)
2. Multiplying a row by a non-zero number (this corresponds to multiplying one of the equations by that number)
3. Replacing a row with a sum of it and another row (this corresponds to adding two equations)

## Example

- Solve the system of equations:

$$2x + y + 2z = 2$$

$$x - y - 4z = 3$$

$$3x + 2y - 2z = -1$$