Review of Basic Algebra

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Sets

- A **Set** is a collection of objects
- To denote a set, braces is used to enclosed a list of its **members** or **elements**.
- To show a set with elements a, b and c
  - { a, b, c }
- To show b is an element of this set
  - b ∈ {a, b, c} (∈ *means* **is an element of**)
- To show d is not an element of this set
  - d∉ {a, b, c} (∉ *means* **is not an element of**)

Set-builder Notation

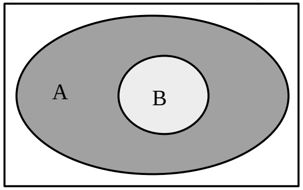
- In Set-builder Notation, a rules is given to establish membership in a set.
- Example:
  - A = { x | x is a odd number of all numbers}
    - A is the set of all number x such that x represents an odd number. (A = 1, 3, 5, ….)
    - Since x can represent many different elements of the set, x is called **variable**.

Equality of Sets

- Given two sets A and B are equal when they have exactly the same elements.
- If set A and B are equal, we write A = B.
- If two set A and C are not equal, we write A ≠ C.
- Example:
  - A = {1, 2, 3}
  - B = {1, 2, 3}
  - C = {1, 2}
  - Therefore, A = B but A ≠ C

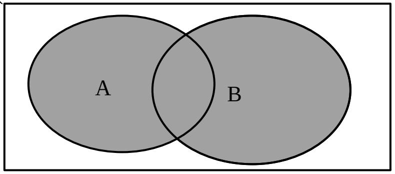
Subsets of Sets

- Given two set A = {a, b, c, d, e} and B = {a, d}.
- B is a subset of A when each elements of B is also an elements of A.
  - B ⊆ A (⊆ *means* **is a subset of**)



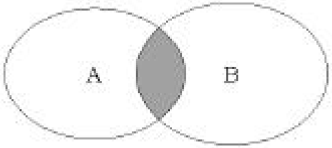
Union of Sets

- If the elements of some set A are joined with the elements of some set B, the union of set A and set B is formed.
- The union of set A and set B is denoted as
  - A ∪ B (∪ *means* **the union of set A and set B**)



Intersection of Sets

- The set of elements that are common to set A and set B is called intersection of set A and set B.
- The intersection of set A and set B is denoted as
  - A ∩ B (∩ *means* **the intersection of set A and set B**)



Empty Set

- A set with no elements is called **Empty Set (**or **Null Set**)
  - It is denoted as ϕ
  - ϕ = { }
- Some Useful Formula
  - A ⊆ A
  - A ∪ ϕ = A
  - A ∩ ϕ = ϕ

Class Exercise

- Given A = {a ,e, i, o, u} and B = {a, b, c, d, e}. Find out the answer of following questions:
  - $A \cup B$
  - $A \cap B$
  - $(A \cap \phi) \cup B$

Natural Numbers

- **Natural numbers** are the counting number and the set of Natural Numbers is the set {1, 2, 3, … }.
- A **Prime Number** is a natural number greater than 1 that is divisible only by itself and 1.
- A **Composite Number** is a natural Number greater than 1 that is not a prime number.
- Note that 1 is the only natural number that is neither prime number nor composite number

Integers

- **Whole Numbers** is the union of 0 and the set of natural number. The set of whole number is the set {0, 1, 2, 3, … }.
- **Integer** is the union of the set of whole numbers and the set of negative numbers. The set of Integer is the set {…, -3, -2, -1, 0, 1, 2, 3, … }.

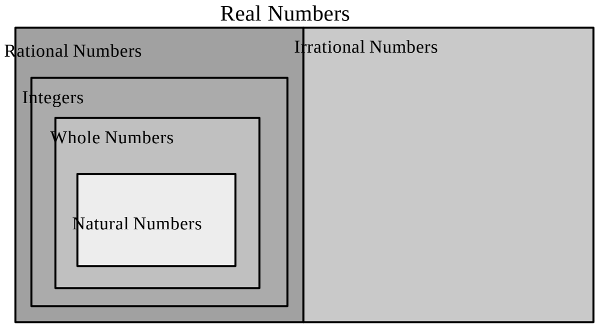
Rational Numbers

- A **Rational Number** is any number that can be written in the form a/b, where a and b are integers and $b \neq 0$. The Integer a is called the ***Numerator***, and the integer b, which cannot be 0, is called ***Denominator***.
- The Rational Numbers are the numbers in the set $\{x \mid x = a/b$, where a, $b \neq 0$ are integer$\}$

Real Numbers

- An **Irrational Number** is a decimal that cannot be written in the form a/b, where a and b are integers and $b \neq 0$.
- **Real Number** is the union of the set of rational numbers and the set of irrational numbers. The set real number is denoted by the symbol $\Re$.

Number System



Class Exercise

- List the numbers in the set {-3, 4/3, 0.12, π, 10}, that are
  - Natural Number
  - Integer
  - Rational Number
  - Irrational Number
  - Real Number

Approximation

- **Truncation** drop all the digits that follow the specified final digits in the decimal.
  - Example: Truncate 20.98765 to two decimal places is 20.98
- **Rounding** identify the specified final digit in the decimal. If the next digit is 5 or more, add 1 to the final digit; if the next digit is 4 or less, leave the final digit as it is. Then truncation the final digit.
  - Example: Round 20.98765 to two decimal places is 20.99

Properties of Real Numbers

- The **Reflexive Property** states that a number always equal to itself, that is a = a.
  - E.g.: x – 4 = x – 4
- The S**ymmetric Property** states that if a = b, then b = a.
  - E.g.: If 5 + x = 3 + y, then 3 + y = 5 + x
- The **Transitive Property** states that if a = b and b = c, then a = c.
  - E.g.: If 3 – x = 8 and 8 = 2 + y, then 3 – x = 2 + y

Principle of Substitution

- The **Principle of Substitution** states that if a and b are real number and a = b, then b can be substituted for a in any mathematical expression to obtain an equivalent expression.
  - Example:
    - If x + 3 = x · y and x = 9, then 9 + 3 = 9 · y

The Closure Properties

- If a and b are real number, then
  - a + b is a real number
  - a - b is a real number
  - a · b is a real number
  - a / b is a real number, provide that b ≠ 0

The Associative Properties

- Associative Property of Addition
  - If a, b, c are real numbers, then
    - a + (b + c) = (a + b) + c = a + b + c
- Associative Property of Multiplication
  - If a, b, c are real numbers, then
    - a · (b · c) = (a · b) · c = a · b · c

The Commutative Properties

- Commutative Property of Addition
  - If a and b are real numbers, then
    - $a + b = b + a$
- Commutative Property of Multiplication
  - If a and b are real numbers, then
    - $a \cdot b = b \cdot a$

The Distributive Property

- The Distributive Property
  - If a, b, c are real numbers, then
    - $a \cdot (b + c) = a \cdot b + a \cdot c$
- The Extended Distributive Property
  - If a, b, c,… are real numbers, then
    - $a \cdot (b + c + d + \ldots) = a \cdot b + a \cdot c + a \cdot d + \ldots$

The Identity Properties

- The Identity Properties for Addition
  - There is a single number 0, called the **Additive Identity**, such that
    - $0 + a = a + 0 = a$
- The Identity Properties of Multiplication
  - There is a single number 1, called the **Multiplicative Identity**, such that
    - $1 \cdot a = a \cdot 1 = a$

The Additive Inverse Property

- If the sum of two numbers is 0, those numbers are called ***additive inverse***, the ***negative***, or the ***opposite*** of each other.
- For each number a, there is a single number -a such that
  - $a + (-a) = -a + a = 0$
- The number –a is called the additive inverse, the negative, or the opposite of a. Also, a is called additive inverse, the the negative, or the opposite of –a

The Multiplicative Inverse Property

- If the Product of two number is 1, the numbers are called ***multiplicative inverse***, or the ***reciprocal*** of each other.
- For every non-zero real number a, there exists a single real number 1/a such that
  - $a \cdot 1/a = 1/a \cdot a = 1$
- The number 1/a is called the multiplicative inverse, or the reciprocal, of a. Also a is called the multiplicative inverse, or the reciprocal, of 1/a.

Class Exercise

- Prove that (a + b) + c = a + (c + b)

Difference and Quotient

- The Difference a – b, also read as "a less b" or "a minus b", is defined as

  $$a - b = a + (-b)$$

- If b is a non-zero real number, the quotient a/b, also read as "a divided by b" or "the ratio of a to b", is defined as

  $$\frac{a}{b} = a \cdot \frac{1}{b} \quad \text{if } b \neq 0$$

Working with Difference and Quotient

- Multiplication by Zero

  $$a \cdot 0 = 0$$

- Division Properties

  $$\frac{0}{a} = 0 \qquad \frac{a}{a} = 1 \qquad \text{if } a \neq 0$$

Rules of Signs

- Six Rules of Signs

$$a(-b) = -(ab) \qquad (-a)b = -(ab) \qquad (-a)(-b) = ab$$
$$-(-a) = a \qquad \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b} \qquad \frac{-a}{-b} = \frac{a}{b}$$

- Example:
  - 2(-3) = - (2 · 3) = -6
  - (-3)(-5) = 3 · 5 = 15

Cancellation Properties

- Cancellation Properties

$$ac = bc \text{ implies } a = b \qquad \text{if } c \neq 0$$
$$\frac{ac}{bc} = \frac{a}{b} \qquad \text{if } b \neq 0, c \neq 0$$

- Zero-Product Property

If $ab = 0$, then $a = 0$ or $b = 0$, or both.

Arithmetic of Quotients

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd} \qquad \text{if } b \neq 0, d \neq 0$$
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \text{if } b \neq 0, d \neq 0$$
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \qquad \text{if } b \neq 0, c \neq 0, d \neq 0$$

Factor and Multiple

- Let a,b,c be natural numbers. If ab=c, then a and b is called a **Factor** of c and c is called a **Multiple** of a or b.
- If a and b are prime numbers, they are called the P**rime Factor** of c. the process of finding factors of a number is called **Factorization**.
- Example:
  - 3 is a factor of 6 and 6 is a multiple of 2

Least Common Multiple

- **Least Common Multiple** (L.C.M.) of several numbers is the least common multiple of these numbers.
- Example: $6 = 2 \times 3$
  $12 = 2 \times 2 \times 3$
  $24 = 2 \times 2 \times 2 \times 3$
  - L.C.M. of 6, 12 and 24 is $2 \times 2 \times 2 \times 3 = 24$.

Highest Common Factor

- **Highest Common Factor** (H.C.F.) of several numbers is the greatest common factor of these numbers.
- Example: $6 = 2 \times 3$
  $12 = 2 \times 2 \times 3$
  $24 = 2 \times 2 \times 2 \times 3$
  - H.C.F. of 6, 12 and 24 is $2 \times 3 = 6$.

Property of HCF and LCM

- Given any two real numbers a and b,
  (H.C.F. of a and b) ´ (L.C.M. of a and b) = ab
- Example:
  - If a = 9, b = 12
    - H.C.F. = 3
    - L.C.M. = 36
    - H.C.F. × L.C.M. = $3 \times 36 = 108$
    - ab = $9 \times 12 = 108$