# Ma104: Mathematics for Business Major

## Assignment 2 (Answer)

**1. Find the slope and distance between the points (3, –2) and (7, 1). (10 marks)**

$$Slope = \frac{(-2)-1}{3-7} = \frac{3}{4}$$

$$Distance = \sqrt{(3-7)^2 + [(-2)-1]^2} = 5$$

**2. Fine an equation of the line containing the point (1, -2) that is perpendicular to the line x + 3y =6. (10 marks)**

We first write the equation of the given line in slope–intercept form to find its slope.

$$\begin{aligned} x + 3y &= 6 \\ 3y &= -x + 6 && \text{Proceed to solve for } y. \\ y &= -\tfrac{1}{3}x + 2 && \text{Place in the form } y = mx + b. \end{aligned}$$

The given line has slope $-\frac{1}{3}$. Any line perpendicular to this line will have slope 3. Because we require the point $(1, -2)$ to be on this line with slope 3 we use the point–slope form of the equation of a line.

$$y - (-2) = 3(x - 1) \qquad \text{Point–slope form}$$

To obtain other forms of the equation we proceed as follows:

$$\begin{aligned} y + 2 &= 3(x - 1) \\ y + 2 &= 3x - 3 \\ y &= 3x - 5 && \text{Slope–intercept form} \\ 3x - y &= 5 && \text{General form} \end{aligned}$$

**3. Using the augment matrix and Gaussian elimination to solve the system of equation (20 marks):**

**x – y + 5z = -6**

**3x + 3y – z = 10**

**x + 3y + 2z = 5**

Since the system is in proper form, begin by writing the augmented matrix of the linear system.

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

the result to the corresponding element in the second row, using transformation 3.

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ \mathbf{0} & \mathbf{6} & \mathbf{-16} & \mathbf{28} \\ 1 & 3 & 2 & 5 \end{array}\right] \quad -3R1 + R2$$

Now, to change the last element in the first column to 0, use transformation 3 and multiply each element of the first row by $-1$, then add the results to the corresponding elements of the third row.

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ \mathbf{0} & \mathbf{4} & \mathbf{-3} & \mathbf{11} \end{array}\right] \quad -1R1 + R3$$

The same procedure is used to transform the second and third columns. For both of these columns perform the additional step of getting 1 in the appropriate position of each column. Do this by multiplying the elements of the row by the reciprocal of the number in that position.

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ \mathbf{0} & \mathbf{1} & \mathbf{-8/3} & \mathbf{14/3} \\ 0 & 4 & -3 & 11 \end{array}\right] \quad \tfrac{1}{6}R2$$

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{7/3} & \mathbf{-4/3} \\ 0 & 1 & -8/3 & 14/3 \\ 0 & 4 & -3 & 11 \end{array}\right] \quad R2 + R1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 7/3 & -4/3 \\ 0 & 1 & -8/3 & 14/3 \\ \mathbf{0} & \mathbf{0} & \mathbf{23/3} & \mathbf{-23/3} \end{array}\right] \quad -4R2 + R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 7/3 & -4/3 \\ 0 & 1 & -8/3 & 14/3 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \end{array}\right] \quad \tfrac{3}{23}R3$$

$$\left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ 0 & 1 & -8/3 & 14/3 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad -\tfrac{7}{3}R3 + R1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \tfrac{8}{3}R3 + R2$$

The linear system associated with this final matrix is

$$\begin{aligned} x &= 1 \\ y &= 2 \\ z &= -1, \end{aligned}$$

and the solution set is $\{(1, 2, -1)\}$. ◗

**4. Graph the compound inequality y > x – 3 and 2y < -x + 4. (10 marks)**

We first graph the equations $y = x - 3$ and $y = -\frac{1}{2}x + 2$. These lines divide the plane into four regions as shown in Fig. 3.29(a). Now test one point of each region to determine which region satisfies the compound inequality. Test the points (3, 3), (0, 0), (4, −5), and (5, 0):

$3 > 3 - 3$ and $3 < -\frac{1}{2} \cdot 3 + 2$ Second inequality is incorrect.

$0 > 0 - 3$ and $0 < -\frac{1}{2} \cdot 0 + 2$ Both inequalities are correct.

$-5 > 4 - 3$ and $-5 < -\frac{1}{2} \cdot 4 + 2$ First inequality is incorrect.

$0 > 5 - 3$ and $5 < -\frac{1}{2} \cdot 0 + 2$ Both inequalities are incorrect.