

Ma104: Mathematics for Business Major

Assignment 1 (Answer)

1. Define which elements of the set $A = \left\{ -\sqrt{10}, -3, -\frac{5}{2}, -0.025, 0, \sqrt{2}, 3\frac{1}{2}, \frac{8}{2} \right\}$ are

members of the following sets. (12 Marks)

A) Natural Number

$$\left\{ \frac{8}{2} \right\}$$

B) Whole Number

$$\left\{ 0, \frac{8}{2} \right\}$$

C) Integer

$$\left\{ -3, 0, \frac{8}{2} \right\}$$

D) Rational Number

$$\left\{ -3, -\frac{5}{2}, -0.025, 0, 3\frac{1}{2}, \frac{8}{2} \right\}$$

E) Irrational Number

$$\left\{ -\sqrt{10}, \sqrt{2} \right\}$$

F) Composite Number

$$\left\{ \frac{8}{2} \right\}$$

2. Let p and q be the roots of the equation $3x^2 - 8x + 2 = 0$.

A) Find $p^2 - q^2$ (7 Marks)

$$p + q = \text{sum of roots} = -\left(\frac{-8}{3}\right) = \left(\frac{8}{3}\right)$$

$$pq = \text{product of roots} = \frac{2}{3}$$

$$p^2 - q^2 = (p + q)(p - q)$$

$$\begin{aligned}
&= (p+q) \left[\sqrt{(p-q)^2} \right] \\
&= (p+q) \left[\sqrt{p^2 - 2pq + q^2} \right] \\
&= (p+q) \left[\sqrt{p^2 + 2pq + q^2 - 4pq} \right] \\
&= (p+q) \left[\sqrt{(p+q)^2 - 4pq} \right] \\
&= \left(\frac{8}{3} \right) \left[\sqrt{\left(\frac{8}{3} \right)^2 - 4 \left(\frac{2}{3} \right)} \right] \\
&= \left(\frac{8}{3} \right) \left[\sqrt{\left(\frac{40}{9} \right)} \right] \\
&= \left(\frac{8}{3} \right) \left[\frac{2\sqrt{10}}{3} \right] \\
&= \frac{16\sqrt{10}}{9}
\end{aligned}$$

B) Find the quadratic equation where the roots are $\frac{p}{q}$ and $-\frac{q}{p}$. (6 Marks)

The roots of the required equation is $\frac{p}{q}$ and $-\frac{q}{p}$, and for this equation

$$\begin{aligned}
\text{Product of roots} &= \left(\frac{p}{q} \right) \left(-\frac{q}{p} \right) \\
&= -1
\end{aligned}$$

$$\begin{aligned}
\text{Sum of roots} &= \frac{p}{q} - \frac{q}{p} \\
&= \frac{p^2 - q^2}{pq} \\
&= \frac{16\sqrt{10}}{9} \\
&= \frac{2}{3} \\
&= \frac{8\sqrt{10}}{3}
\end{aligned}$$

\ The required equation is

$$\begin{aligned}
x^2 - \frac{8\sqrt{10}}{3}x - 1 &= 0 \\
3x^2 - 8\sqrt{10}x - 3 &= 0
\end{aligned}$$

3. Find the real solutions of the equation $(x^2 - 1)^2 + (x^2 - 2) - 12 = 0$. (5 Marks)

Let $u = x - 1$, there the equation become $u^2 + u - 13 = 0$

$$u = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-13)}}{2(1)}$$

$$u = \frac{-1 + \sqrt{53}}{2} \quad \text{or} \quad u = \frac{-1 - \sqrt{53}}{2} \text{ (rejected)}$$

$$x^2 - 1 = \frac{-1 + \sqrt{53}}{2}$$

$$x^2 = \frac{-1 + \sqrt{53}}{2} + 1$$

$$x^2 = \frac{1 + \sqrt{53}}{2}$$

$$x = \sqrt{\frac{1 + \sqrt{53}}{2}} \quad \text{or} \quad x = -\sqrt{\frac{1 + \sqrt{53}}{2}}$$

4. John and Mary are traveling to a business conference. John travels 110km in the same time Mary travels 140km. Mary travels 15Km/hr faster than John. Find the average rate of each person. (8 marks)

Let x = John's speed, then

$x + 15$ = Mary's speed.

	<i>Distance</i>	<i>Speed</i>	<i>Time</i>
<i>John</i>	110Km	x	$\frac{100}{x}$
<i>Mary</i>	140km	$x + 15$	$\frac{100}{x + 15}$

Since they are "in the same time", so John's time = Mary's time.

$$\frac{110}{x} = \frac{140}{x + 15}$$

$$110(x + 15) = 140x$$

$$110x + 1650 = 140x$$

$$1650 = 30x$$

$$x = 55$$

Therefore, John's speed is 55km/hr and Mary's speed is 70km/hr.

5. A company charges \$200 for each box of tools on order of 150 or fewer boxes. If a customer orders x boxes in excess of 150, the cost of each box ordered is reduced to x dollars. If David's bill came to \$30,625, how many boxes were ordered? (6 marks)

If the customer orders 150 boxes, the total cost will be \$30,000. Therefore the customer ordered over 150 boxes.

Since the price will reduce to x when over 150 boxes, then

$$x^2 = 30625$$

$$x = 175$$

The customer ordered 175 boxes.

- 6. Normal human body temperature is 98.6°F . If a temperature x that differs from normal by at least 1.5°F is considered unhealthy. Write the condition for an unhealthy temperature x as an equality involving an absolute value, and solve for x (6 marks)**

$$|x - 98.6| \geq 1.5$$

$$x - 98.6 \leq -1.5 \text{ or } x - 98.6 \geq 1.5$$

$$x \leq 97.1 \text{ or } x \geq 100.1$$