Set Theory

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What is Set?

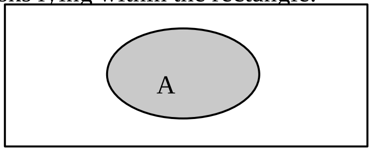
- A **Set** is any well-defined list, collection, or class of objects.
- The objects in set can be anything
- These objects are called the **Elements** or **Members** of the set.

Notation

- Tabular Form
  - A = {a, e, i, o, u}
- Set-builder Form
  - B = {x | x is odd}
- (Do Ex. 1 – 10)

Venn Diagram

- A **Venn Diagram** is a pictorial representation of sets by set of points in the plane.
- The Universal Set U is represented by the interior of a rectangle, and the other sets are represented by disks lying within the rectangle.



Type of Sets

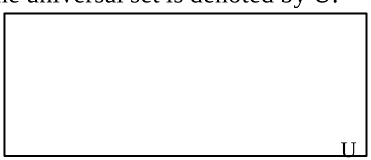
- Finite
- Infinites Set
- Universal Set
- Subset
- Proper Subset
- Null Set
- Disjoint Set
- Sets of Sets
- Power Sets

Finite and Infinite Set

- A **Finite Set** is a set consisting specific number of different elements.
  - Let X = {1, 2, 3, 4, …. 100}, then X is finite.
- A **Infinite Set** is a set consisting infinite number of different elements.
  - Let X = {1, 3, 5, 7, …..}, then X is infinite.

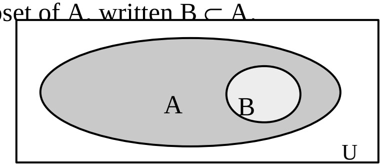
Universal Set

- All the set under consideration can thought of as subsets or another set called the **Universal Set**.
- The universal set is denoted by U.



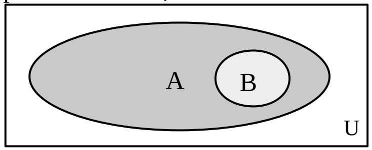
Subset

- B is a **Subset** of A if every element of B is also an element of A.
  - Example: Given Set A = {a, b, c, d, e}, Set B = {a, d}. B is a subset of A when each elements of B is also an elements of A.
- If B is subset of A, written $B \subset A$.



Proper Subset

- B is a **Proper Subset** of A if B is a subset of A and B is not equal to A.
  - Example: Given Set A = {a, b, c, d, e}, Set B = {a, d}, Set C = {a, d}. Then B is a proper subset of A, but B is not a proper subset of C.
- If B is proper subset of A, written $B \subset A$.

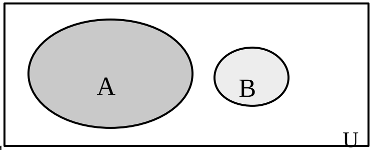


Null Sets

- A set with no elements is called **Empty Set (**or **Null Set**).
- The Null Set ϕ is a subset of every set.
- The Empty Set is denoted by ϕ.
- Example:
  - Let B = $\{ x \mid x^2 = 4, x \text{ is odd}\}$, then B = ϕ.

Disjoint Sets

- Two Sets A and B are called **Disjoint Sets** if they have nothing in common.
  - Example: Let A = {a, b}, B = {c, d}, then A and B are disjoint sets.
- In Set Notation, $A \cap B = \phi$.



Power Sets

- The **Power Sets** of a set X, is denoted by P(X), is the set of all subset of X.
- If a finite set X has n elements, then power set of X contain 2^n elements.
- Example:
  - Let Set X = {0, 1},
    then P(X) = $\{\phi, \{0\}, \{1\}, \{0, 1\}\}$.

Sets of Sets

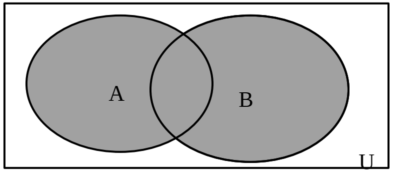
- Sets can be the elements of another sets.
- We called them **Sets of Sets**, **Family of Sets** and **Class of Sets**.
- The set A = {{2, 3}, {2}, {5, 6}} is a family of sets. Its members are the sets {2, 3}, {2}, {5, 6}.

Operation of Sets

- Union
- Intersection
- Difference
- Complement

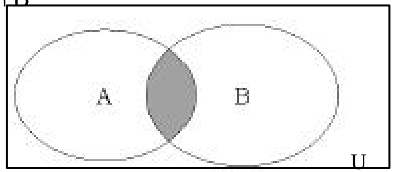
Union of Sets

- If the elements of some set A are joined with the elements of some set B, the **Union** of set A and set B is formed.
- The union of set A and set B is denoted as $A \cup B$.



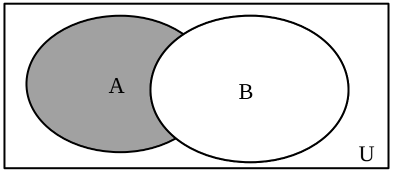
Intersection of Sets

- The set of elements that are common to set A and set B is called **Intersection** of set A and set B.
- The intersection of set A and set B is denoted as $A \cap B$



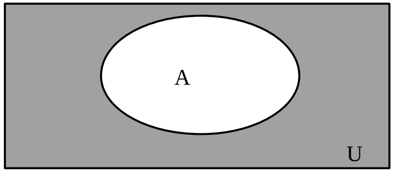
Difference

- The **Difference** of Set A and B is the set of elements which belong to A but don't belong to B.
- The different of set A and set B is denoted as A – B, A \ B, A ~ B.



Complement

- The Complement of Set A is the Set of elements which do not belong to A.
- The Complement of A is denoted by A', $\bar{A}$, A^c. (Do Ex. 11 – 16)



Attributes of Sets

- Equality of Sets
- Comparability
- Cardinality
- Principle of Inclusion and Exclusion

Equality of Sets

- Set A is equal to set B is they both contains the same elements.
- The equality of A and B is denoted by A = B.
- We can also say A = B if and only if $A \subseteq B$ and $B \subseteq A$.

Comparability

- Two sets A and B are said to be **Comparable** if one of the sets is a subset of the other set.
- Two set are said to be **Incomparable** if neither one of the set is a subset of the others.

Cardinality

- The number of elements in a set is called its **Cardinality**. Cardinality is denoted by |.
- Example:
  - $|\{1, 3, 5, 7, 9\}| = 5$

Principle of Inclusion and Exclusion

- If A and B be finite sets, then $|A \cup B| = |A| + |B| - |A \cap B|$
- Example:
  - Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4\}$
  - Then $|A \cup B| = 5$, $|A| = 3$, $|B| = 4$, $|A \cap B| = 2$; $|A| + |B| - |A \cap B| = 3 + 4 - 2 = 5$.

Proofs

- Using Venn Diagrams (Do Ex. 17 – 19)
- Proof using Algebra Laws (Do Ex. 20 – 21)
- Three Set Venn Diagram (Do Ex. 22)
- Venn Diagrams using Regions (Do Ex. 23 – 30)