Introduction to Logic

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What is Logic?

- Logic is the study of reasoning
- It is specifically concerned with whether reasoning is correct
- Logic is also known as **Propositional Calculus**

Simple Statements

- Simple Statement is the basic building block of Logic.
- Simple Statement is referred as a proposition.
- A statement is a declarative sentence that either True or False.

Which one is statement?

- Today is Friday.
  - This is a Statement
- How to celebrate the Mid-Autumn?
  - This is not a Statement
- Let's go for dinner together after lesson.
  - This is not a Statement
- 1 + 1 = 3
  - This is a Statement

Compound Statement

- **Compound Statement** is the combination of two or more Simple Statement.
- Example:
  - “Today is Friday” and “Tomorrow is holiday”

True Table

- The value of a statement can be represented by a Truth Table.
- Only **True** and **False** is appear in a Truth Table
- Example:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Basic Logic Connectives

- Compound statements are connected using mainly five basic connectives:
  - Conjunction
  - Disjunction
  - Negation
  - Conditional
  - Biconditional

Conjunction

- Conjunction is the combination of statements using **AND**.
- The conjunction of two statement is True only if each component is True.
- Represented as p ^ q.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (Inclusive OR)

- Disjunction is the combination of statements using **OR**.
- The conjunction of two statement is True if either one component is True.
- Represented as p∨ q.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Negation

- Negation is the **NOT** of a simple statement.
- The Truth value of the statement negation of a statement is the opposite of the truth value of the original statement.
- Represented as ~p.

p	~p
T	F
F	T

Conditional

- Conditional Statement is the statement in the form "If p, then q" or "p implies q".
- The conditional p → q is True unless p is True and q is False.
- Represented as p → q
- (Do the Ex. 1 & 2)

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

- Biconditional Statement is the statement in the form "p if and only if q" or "p iff q".
- If p and q have the same value, p ↔ q is True, otherwise will be False.
- Represented as p ↔ q

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Propositions

- When the sub-statement of a compound statement are variables and represented in logical connectives, the compound statement is called Proposition.

Truth Table

- The truth value of a proposition depends exclusively upon the truth values of its variables.
- The truth value of a proposition is known once the truth values of its variables are known.
- (See E.g. 30 - 32)

Exclusive Disjunction (Exclusive OR)

- Exclusive Disjunction means "either one or the other, but not both".
- The conditional of a exclusive disjunction is True when p and q are not the same.
- Exclusive Disjunction can be expressed using basic connectives $\sim (p \leftrightarrow q)$
- (Do the Ex. 3)

Tautologies and Contradictions

- Tautologies
  - A Compound statement that is always True is called Tautologies
  - (See E.g. 33)
- Contradictions
  - A Compound statement that is always False is called Contradictions
  - (See E.g. 34)

Principle of Substitution

- If P(p, q, …) is a Tautology, then P(P_1, P_2, …) is also a Tautology.
- (See E.g. 35)

Law of Syllogism

- A fundamental principle of logical reasoning, called the Law of Syllogism, states “If p implies q and q implies r, then p implies r”
- $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology
- (See E.g. 36)

Logical Equivalence

- Two propositions P and Q are said to be logically equivalent if the final columns in their truth table are the same.
- Represented as º
- (Do the Ex. 4 & 5)

DeMorgan’s Laws

- DeMorgan’s Laws show that:
  - $\sim (p \vee q) \equiv \sim p \wedge \sim q$
  - $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Logically True & Logically Equivalent

- Logically True Statement
  - A statement is said to be Logically True if it is derivable from a Tautology.
- Logically Equivalent Statement
  - Statement of the form $P(p_0, q_0, ..)$ and $Q (p_0, q_0, ..)$ are said to be Logically Equivalent if the propositions P(p, q, …) and Q(p, q, ..) are logical equivalent.

Argument

- An argument is a relationship between a set of proposition, $P_1, P_2, \ldots P_n$, called **Premises** and other proposition Q, called the **Conclusion**.
- An argument is denoted by $P_1, P_2, \ldots P_n \vdash Q$
- An argument is said to be **Valid** if the premises yield the conclusion.
- An argument is said to be **Fallacy** if that is not valid.
- (See E.g. 39 & 40)

Logical Implication

- A proposition P(p, q, …) is said to Logical Imply a proposition Q(p, q, …), written P(p, q, …) => Q(p, q, …) if Q(p, q, …) is true whenever P(p, q,) is True.
- (See E.g. 44)

Example

- Consider the following argument: if I am not in Malaysia, then I am not happy; if I am happy, then I am singing; I am not singing; *therefore* I am not in Malaysia. Using the translation

h	I am happy
m	I am in Malaysia
s	I am singing

show that this argument is valid, or explain why it is not.

Answer

h	m	s	$\sim m \to \sim h$	$h \to s$	$\sim s$	$(\sim m \to \sim h) \wedge (h \to s) \wedge (\sim s)$	$(\sim m \to \sim h) \wedge (h \to s) \wedge (\sim s) \to \sim m$
T	T	T	T	T	F	F	T
T	T	F	T	F	T	F	T
T	F	T	F	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	F	F	T
F	T	F	T	T	T	T	F
F	F	T	T	T	F	F	T
F	F	F	T	T	T	T	T

This is *not* a valid argument.

Laws of Algebra of Propositions

- Idempotent Laws
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Associative Laws
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Communicative Laws
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$
  - $(p \leftrightarrow q) \equiv (q \leftrightarrow p)$

Laws of Algebra of Propositions

- Distributive Laws
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- Identity Laws
  - $p \vee F \equiv p$
  - $p \wedge T \equiv p$
  - $p \vee T \equiv T$
  - $p \wedge F \equiv F$

  (T = True, F = False)

Laws of Algebra of Propositions

- Complement Laws
  - $p \vee \sim p \equiv T$
  - $p \wedge \sim p \equiv F$
  - $\sim T \equiv F$
  - $\sim F \equiv T$

  (T = True, F = False)
- Involution Law
  - $\sim \sim p \equiv p$

Laws of Algebra of Propositions

- DeMorgan's Laws
  - $\sim(p \lor q) \equiv \sim p \land \sim q$
  - $\sim(p \land q) \equiv \sim p \lor \sim q$
  - $(p \lor q) \equiv \sim(\sim p \land \sim q)$
  - $(p \land q) \equiv \sim(\sim p \lor \sim q)$
- Contrapositive
  - $(p \to q) \equiv (\sim q \to \sim p)$

Laws of Algebra of Propositions

- Implication
  - $(p \to q) \equiv (\sim q \lor p)$
  - $(p \to q) \equiv \sim(p \land \sim q)$
  - $(p \lor q) \equiv (\sim p \to q)$
  - $(p \land q) \equiv \sim(p \to \sim q)$
  - $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
  - $(p \to q) \land (p \to r) \equiv p \to (q \land r)$
- Equivalence
  - $(p \leftrightarrow q) = (p \to q) \land (q \to p)$

Laws of Algebra of Propositions

- Exportation Law
  - $(p \land q) \to r \equiv p \to (q \to r)$
- Absorbtion Law
  - $(p \lor q) \land (p \land q) \equiv p \lor q$
  - $(p \land q) \land (p \lor q) \equiv p \lor q$
- Reductio ad adsurdum
  - $(p \to q) \equiv (p \land \sim q) \to F$

  (T = True, F = False)