

Question 1 (Compulsory)**(30 marks)**

- (a) Let the sets A , B , C , D and E be given by

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{1, 3, 5, 7, 9\}$$

$$D = \{3, 4, 5\}$$

$$E = \{3, 5\}$$

Which of the sets above could be the set X in the following cases?

- (i) $X \subset C$ but $X \not\subset B$

- (ii) X and B are disjoint [2]

- (b) Given the statement “If the liquid is boiling, then its temperature is 120°C ” determine whether or not each of the following statements are true.

- (i) The liquid is not boiling or the temperature of the liquid is 120°C .

- (ii) The temperature of the liquid is 120°C if and only if the liquid is not boiling. [2]

- (c) Let p and q be any two statements. Construct a truth table for the compound statement

$$[(p \leftrightarrow q) \wedge q] \rightarrow p \quad [4]$$

- (d) Let A and B be events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$ and $P(A^c) = 5/8$.

- (i) Calculate $P(A)$. [1]

- (ii) Calculate $P(B)$. [2]

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- (e) Let the matrices A , B and C be given by

$$A = \begin{pmatrix} -2 & x \\ 6 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 6 \\ 15 & 10 \end{pmatrix}$$

- (i) Calculate the matrix product AB . [2]
- (ii) If $AB=C$ calculate the value of x . [2]
- (f) Draw four graphs that each have four vertices, at least four edges and are not isomorphic. [4]
- (g) Prove by mathematical induction that $2^n < n!$ for $n = 4, 5, 6, 7, \dots$ [5]
- (h) Let the sets A and B be given by $A = \{a, b, c\}$ and $B = \{1, 2\}$.
- (i) Write down the Cartesian product $A \times B$. [2]
- (ii) How many relations are there between A and B ? Justify your answer. [2]
- (iii) How many functions are there from A to B ? Justify your answer. [2]

Please turn over

Question 2

- (a) Briefly explain the difference between a relation and a function. [2]
- (b) Consider the following relations R , S and T on the set $A = \{1, 2, 3\}$.
- (i) Is $R = \{(1,1), (1,2), (2,2), (2,3)\}$ a transitive relation? [2]
- (ii) Is $S = \{(1,1), (2,2)\}$ an anti-symmetric relation? [2]
- (iii) Is $T = \{(1,1), (3,3)\}$ an equivalence relation? [2]

Justify your answers.

(max 6 marks)

- (c) A sandwich shop stocks 3 varieties of bread and 3 varieties of filling for sandwiches. A sandwich consists of one type of bread and one type of filling. How many different types of sandwich can be bought from this shop? [2]
- (d) A café serves 10 different varieties of coffee. In a promotion the café has distributed vouchers to customers that entitle them to 4 free cups of coffee.
- (i) In how many different combinations can the 4 cups of coffee be chosen if each variety of coffee may be ordered only once? [2]
- (ii) In how many different combinations can the 4 cups of coffee be chosen if any variety of coffee may be ordered as many times as desired? [3]

Please turn over

Question 3

- (a) Let the sets A , B , C and D be defined as follows.

A : Expensive trees
 B : Trees planted last year
 C : Fruit trees
 D : Trees in the orchard

These sets satisfy the following conditions:

S_1 : All expensive trees were planted last year
 S_2 : All fruit trees are in the orchard
 S_3 : No tree in the orchard was planted last year

- (i) Draw a Venn diagram of the sets A , B , C and D that satisfies the conditions S_1 , S_2 and S_3 . Recall that the discs representing two disjoint sets in a Venn diagram will not overlap. [3]

- (ii) Using your Venn diagram from part (i), state which of the statements below are true.

T_1 : A fruit tree was planted last year
 T_2 : No expensive tree is in the orchard
 T_3 : No fruit tree is expensive [3]

- (b) Give counter examples to show that the function $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = x^2 - 10$ is neither a one-to-one function nor an onto function. [4]

- (c) Consider the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x^2 + 1$ and $g(x) = 3x + 4$.

- (i) Find the formula defining the composition $(g \circ f)(x)$ [2]

- (ii) Find the formula for the function $g^{-1}(x)$ [3]

Please turn over

Question 4

- (a) (i) What does it mean to say that two events are independent?
- (ii) What does it mean to say that two events are mutually exclusive?

Do not give mathematical formulae in your answer [2]

- (b) The probability that a man will live for at least 10 more years is $1/4$ and the probability that his wife will live for at least 10 more years is $1/3$. Assume that these events are independent.

Find the probability that

- (i) Both will be alive in 10 years time [1]
- (ii) At least one of them will be alive in 10 years time [2]
- (iii) Only the wife will be alive in 10 years time [3]

- (c) Let U be the universal set, and let A and B be any two sets. Prove the identity below using the laws of set theory. Identify clearly the laws that are used.

$$(U \cap A) \cup (B \cap A) \equiv A \quad [3]$$

- (d) Let p be the statement “he is tall”, let q be the statement “he is handsome” and let r be the statement “he is muscular”. Write each of the following compound statements in symbolic form using p , q , r and logical connectives.

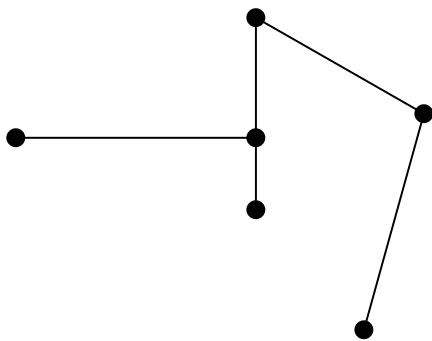
(Note that “he is short” means “he is not tall”)

- (i) If he is tall and muscular then he is handsome. [1]
- (ii) He is neither tall nor handsome [1]
- (iii) He is tall or he is short and handsome [1]
- (iv) It is not true that he is short or not handsome. [1]

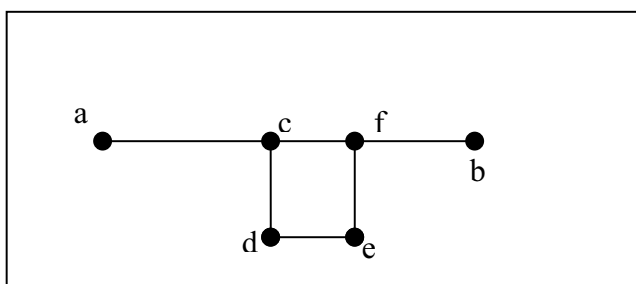
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Question 5

- (a) Let R be the relation on the set $V = \{2, 3, 4, 9, 15\}$ defined by xRy if and only if x is less than y .
- (i) List the members of R . [2]
- (ii) Draw the digraph representing the relation R . [2]
- (b) (i) Draw the graph $K_{2,3}$. [2]
- (ii) Write down the adjacency matrix for the graph $K_{2,3}$. [2]
- (c) Define what is meant by a tree in the context of graph theory. Is the graph below a tree? [3]



- (d) Consider the graph below.



- (i) Is the graph semi-Eulerian? Justify your answer. [2]
- (ii) Is the graph semi-Hamiltonian? Justify your answer. [2]

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