Question 1 (Compulsory)

(a) A bag contains 4 red marbles, 6 blue marbles and 5 green marbles. How many ways are there to choose 2 red, 4 blue and 3 green marbles from the marbles in this bag? [3]

(b) Let R be the relation on the set Z+ of positive integers defined by aRb if and only if $a = b$.

Prove or give a counter-example to determine whether R has the following properties:

(i) reflexivity; [2]

(ii) symmetry; [2]

(iii) transitivity. [2]

(c) The matrices A and B are given by

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 7 & 2 \\ 6 & 3 \end{pmatrix}$$

(i) Calculate AB [2]

(ii) Calculate BA [2]

(iii) Does $(A+B)^2 = A^2 + 2AB + B^2$? Justify your answer. [2]

(d) A function, $f(n)$, defined for positive integers n, satisfies the following conditions:

$$f(1) = 2$$
$$f(n+1) = 2f(n) \quad \text{for } n = 1,2,3,...$$

Use mathematical induction to prove that $f(n) = 2^n$ for $n = 1,2,3,....$ [6]

Question 1 continues on the next page.

Question 1 - continued.

(e) A man throws a die repeatedly until a six appears on the top face. Calculate the probability that:

(i) the first six appears on the second throw; [2]

(ii) the first six appears on the fourth throw given that a six does not appear on the first two throws. [4]

(f) The sets A and B are defined by $A = \{1,2,3,4\}$ and $B = \{x \mid x = y^2, y \in A\}$. Write down the members of the following sets:

(i) B [1]

(ii) $A \cap B$ [1]

(iii) $B - A$ [1]

Question 2

(a) A bit string of length 10 is chosen at random.

(i) How many such bit strings are there? [2]

(ii) In how many different ways can the bit string contain exactly three 0's? [2]

(iii) In how many different ways can the bit string be chosen so that the first five bits contain exactly three 0's and the last five bits contain exactly two 0's? [3]

(iv) In how many different ways can the bit string contain at least two 0's? [3]

(b) Let A and B be two events. You are given the following probabilities:

$P(A)=0.7$ $\quad$ $P(B)=0.5$ $\quad$ $P(A \cup B)=0.8$

Are the events A and B independent? Justify your answer. [5]

Question 3

(a) Consider the following argument:

If the workers are well paid, then the project will be finished on time. If the project is not finished on time, then the boss will be sacked. The boss is not sacked, therefore the workers are well paid.

Defining w, p, b as follows:

w the workers are well paid;
p the project is finished on time;
b the boss is sacked,

construct a truth table to determine whether or not this argument is valid. [8]

(b) Show, using Venn diagrams, that the sets $(A - B) \cap (C - B)$ and $((A \cap C) - B) \cap B'$ are equal. Show your working clearly. [7]

Question 4

(a) (i) What is meant by saying that a function $f : A \to B$ is a 1-1 function? [2]

(ii) What is meant by saying that a function $g : A \to B$ is an onto function? [2]

(b) The functions $f : \Re \to \Re$ and $g : \Re \to \Re$ are given by

$$f(x) = 16 + 5x \qquad g(x) = x^2$$

Note that $\Re$ is the set of real numbers.

(i) Prove that f is a 1-1 function. [2]

(ii) Is g an onto function? Prove or give a counterexample. [2]

(iii) Calculate $f^{-1}(x)$. [3]

(iv) Calculate $g(f(x))$. [2]

(v) Calculate $f(g(f(x)))$. [2]

Question 5

(a) Let G be a graph. Prove that the sum of the degrees of the vertices of G is equal to twice the number of its edges. [2]

(b) Calculate the number of edges of each of the following graphs, if the graph exists. For graphs that do not exist, explain their non-existence.

(i) G1: A 5-regular graph with 7 vertices. [1]

(ii) G2: The complete bipartite graph $K_{2,4}$. [1]

(iii) G3: The complete graph K_5 with 5 vertices. [1]

(c) For the graphs that do exist:

(i) Draw the graphs, labelling all vertices [4]

(ii) Write down their adjacency matrices and their distance matrices. [6]

- END OF PAPER -