Question 1 (Compulsory)

(a) Let A be the set given by $A = \{x \mid x^2 = 4\}$

(i) List the elements of the set A. [1]

(ii) Write down all the subsets of the set A. [2]

(b) Let R be the relation on the set $A = \{1,2,3,4\}$ given by

$R=\{(1,1), (1,3), (3,2), (3,4), (4,2)\}$

(i) Find R^{-1}, the inverse relation of R [1]

(ii) List the ordered pair elements that must be added to R so that it is reflexive. [2]

(c) Construct a truth table to show that, for any two statements p and q, $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ [4]

(d) Let n be any positive integer. Let S be the statement "If n^2 is even then n is even."

(i) Write down the contrapositive of the statement S. [2]

(ii) Use your answer to part (i) to give a contrapositive proof of S. [3]

(e) A bit string of length 5 is generated.

(i) How many different bit strings may be generated? [2]

(ii) How many different bit strings contain three 1s and two 0s? [2]

(iii) What is the probability that the bit string generated contains three 1s and two 0s? [2]

Question 1 continues on the following page.
Please turn over

(f) (i) Write down the transpose of the matrix $A = \begin{pmatrix} 9 \\ -11 \\ 10 \end{pmatrix}$ [1]

(ii) The matrices A and B are given by

$$A = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$

Calculate the matrix product AB. [2]

(g) Draw the following graphs.

(i) K_5 [2]

(ii) C_5 [2]

(iii) $K_{3,2}$ [2]

Please turn over

Question 2

(a) Prove by mathematical induction that

$$\sum_{i=1}^{n} 2i = n(n+1)$$

for $n = 1,2,3,\ldots$ [6]

(b) There are 10 lecturers in the computer science department at Springfield University. The computer science department offers 5 different courses one semester. Each course is taught by one lecturer. If each lecturer teaches at most one course, in how many different ways can the teaching duties be allocated? [2]

(c) How many people are needed to ensure that at least four of them have their birthday in the same month? [2]

(d) A rugby team consists of 8 forward players and 7 back players. Trumpton rugby club has a squad of players consisting of 12 forward players and 10 back players.

(i) In how many different ways can the forward players be chosen? [2]

(ii) In how many different ways can the back players be chosen? [2]

(iii) In how many different ways can the rugby team be chosen? [1]

Please turn over

Question 3

(a) Consider the argument given below.

"If she goes the beach, Victoria will get sunburnt. Victoria will go to the beach if and only if she is unemployed. Victoria is employed. Therefore she will not get sunburnt."

Define the statements *p*, *q* and *r* as follows

p: Victoria goes to the beach
q: Victoria gets sunburnt
r: Victoria is employed

(i) Write down the argument in symbolic form. [3]

(ii) Construct a truth table for the argument. [6]

(iii) Is the argument valid? Justify your answer. [1]

(b) Let the sets A and B be defined by

A={1, 2, 3, 4, 5}
B={x | x is an integer between 4 and 10 inclusive}

Write down the elements of the following sets.

(i) B [1]

(ii) $A \cup B$ [1]

(iii) $A \cap B$ [1]

(iv) A - B [1]

(v) $(A \cap B) \times (A - B)$ [1]

Please turn over

Question 4

(a) Consider the following relations on the set of integers:

$R_1 = \{(a,b) \mid a \geq b\}$
$R_2 = \{(a,b) \mid a > b\}$
$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$
$R_4 = \{(a,b) \mid a = b\}$
$R_5 = \{(a,b) \mid a = b+1\}$
$R_6 = \{(a,b) \mid a + b \leq 3\}$

(i) State which of the relations are reflexive. [2]

(ii) State which of the relations are anti-symmetric. [2]

(iii) State which of the relations are transitive. [2]

(iv) State which of the relations are partial ordering relations. [2]

Justification of answers is **NOT** required.

(b) A fair coin is tossed three times. The number of times that the coin shows heads on these three tosses is recorded.

(i) Calculate the probability that exactly zero heads are seen. [1]

(ii) Calculate the probability that exactly one head is seen. [1]

(iii) Calculate the probability that exactly two heads are seen. [1]

(iv) Calculate the probability that exactly three heads are seen. [1]

(v) Using your answers to parts (i)-(iv) calculate the expected number of heads seen. [3]

Please turn over

Question 5

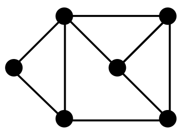
(a) Define a simple graph. [2]

(b) Write down the distance matrix for K_4, the complete graph on four vertices. [2]

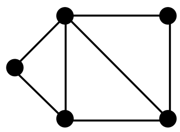
(c) Draw the graph corresponding to the adjacency matrix below. [3]

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

(d) Consider the graphs G and H shown below.



G



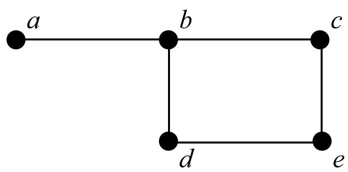
H

(i) Show that the graph G is Hamiltonian by drawing a Hamiltonian cycle. [2]

(ii) Show that the graph H is semi-Eulerian by drawing an Eulerian trail. Be sure to indicate the direction that each edge is traversed with an arrow. [2]

(e) (i) Explain what is meant by a bipartite graph. [2]

(ii) By identifying two suitable sets of vertices, or otherwise, show that the following graph is bipartite: [2]



H

- END OF PAPER -