

### Question 1 (Compulsory)

- (a) The probability that it will rain on Saturday is 0.25, and the probability that it will rain on Sunday is 0.5. Assume that rain on Saturday is independent of rain on Sunday.
- (i) Calculate the probability that it will rain on both Saturday and Sunday. [2]
  - (ii) Calculate the probability that it will rain on at least one of Saturday or Sunday. [2]
- Show your working clearly.
- (b) Give a direct proof to show that if  $n$  is an even integer then  $n^2$  is also an even integer. [3]
- (c) Write the **negation** of the following propositions. [4]
- (i) If the input file exists, then an error message is not generated.
  - (ii) My car is in the workshop and I shall not attend class.
- (d) How many different ways are there to select 6 students from a class of 25 students to serve on a committee? [2]
- (e) How many different ways are there to select 6 students from a class of 25 students to hold 6 different executive positions on a committee? [2]
- (f) How many different ways are there to select 12 objects from objects of 5 different types? [2]
- (g) Using arrow diagrams, draw all the functions from the set  $X = \{a, b\}$  to the set  $Y = \{1, 2\}$ . Which of these functions are invertible? [5]

*Question 1 continues on the following page.  
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- (h) Let the sets  $A$  and  $B$  be given by  $A = \{a, b, c\}$  and  $B = \{c, d\}$ .
- (i) Write down the elements of the set  $A \cup B$
  - (ii) Write down the elements of the set  $A - B$
  - (iii) Write down the elements of the set  $A \times B$  [4]
- (i) Let  $A$  and  $B$  be matrices. The matrix  $B$  has dimension  $5 \times 2$  and the matrix product  $AB$  exists and results in a square matrix. What is the dimension of each following matrices? [4]
- (i)  $AB$
  - (ii)  $B^t$
  - (iii)  $BA$
  - (iv)  $A$

*Please turn over*

**Question 2**

- (a) Consider the following argument.

Peter is a programmer if and only if he has an Advanced Diploma in Computer Studies. He does not have an Advanced Diploma in Computer Studies therefore he is not a programmer.

Define the statements  $p$  and  $q$  by

$p$  – Peter is a programmer

$q$  – Peter has an Advanced Diploma in Computer Studies

- (i) Write the argument in symbolic form. [2]
- (ii) Test the validity of the argument using a truth table. [4]
- (b) Let  $p$  and  $q$  be propositions. Use the laws of logic to determine whether the compound proposition below is a contradiction or a tautology. Identify clearly the laws used. [4]

$$[(p \vee q) \wedge (\sim p)] \rightarrow q$$

- (c) Let the set  $A$  be given by  $A = \{1, 2, 3\}$  and define a relation  $R$  on  $A$  by

for all  $x, y \in A$ ,  $(x, y) \in R$  if and only if  $x \leq y$

- (i) Write down the elements of  $R$ . [1]
- (ii) Is  $R$  symmetric? [2]
- (iii) Is  $R$  transitive? [2]

Justify your answers.

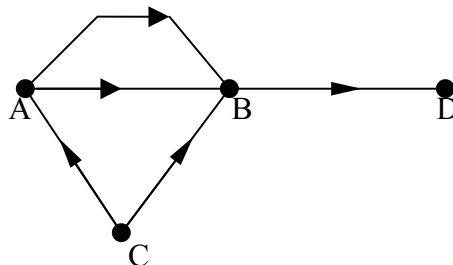
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### Question 3

- (a) (i) Define an Eulerian trail.
- (ii) Define a Eulerian circuit.
- (iii) Draw a graph with 5 vertices that contains an Eulerian circuit. Indicate the Eulerian circuit on your graph.
- (iv) Draw a graph with 5 vertices that contains an Eulerian trail but not an Eulerian circuit. Indicate the Eulerian trail on your graph. [6]
- (b) Draw the graph corresponding to the adjacency matrix below. [2]

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

- (c) Consider the following digraph.



- (i) Write down the distance matrix of the digraph.
- (ii) Write down the in-degree and out-degree of each vertex of the digraph. [4]
- (d) Let the matrices  $A$  and  $B$  be given by

$$A = \begin{pmatrix} -4 & 0 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ 4 & 0 \end{pmatrix}$$

- (i) Calculate  $A+5B$ .
- (ii) Calculate  $AB$ . [3]

*Please turn over*

### Question 4

- (a) The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is given by  $f(x) = 5x - 4$
- (i) Calculate the composite function  $f(f(x))$
  - (ii) Show that  $f$  is an onto function.
  - (iii) Show that  $f$  is a one-to-one function.
  - (iv) Does the inverse function  $f^{-1}(x)$  exist? Justify your answer. [7]
- (b) Suppose 12 people read the Wall Street Journal, Business Week or both. Given that 3 people read only the Wall Street Journal and 6 read both the Wall Street Journal and Business Week, find the number of people who read only Business Week. [2]
- (c) The England football manager is about to choose his squad of 23 players for the World Cup Finals. In the squad he will choose 3 goalkeepers from a list of 4 goalkeepers and 20 outfield players from a list of 28 outfield players.
- (i) In how many different ways can the goalkeepers be selected?
  - (ii) In how many different ways can the outfield players be selected?
  - (iii) In how many different ways can the squad be selected? [6]

*Please turn over*

### Question 5

- (a) Let the set  $A$  be given by  $A = \{a\}$ . Denote the powerset of  $A$  by  $P(A)$ .
- (i) Write down the elements of  $P(A)$
  - (ii) Write down the elements of  $P(P(A))$  [3]
- (b) Prove by mathematical induction that  $5^n - 1$  is divisible by 4 for  $n = 1, 2, 3, \dots$  [5]
- (c) The probability that I solve a programming question correctly is 0.5. I have 5 programming questions to answer.
- (i) What is the probability that I answer all 5 questions correctly?
  - (ii) What is the probability that I answer 4 of the questions correctly and 1 of the questions incorrectly?
  - (iii) What is the probability that I answer at least 2 of the questions incorrectly?

Show your working clearly. [7]

**- END OF PAPER -**